



## Non-Fickian mass transport in fractured porous media

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### ABSTRACT

The paper provides an introduction to fundamental concepts of mathematical modeling of mass transport in fractured porous heterogeneous rocks. Keeping aside many important factors that can affect mass transport in subsurface, our main concern is the multi-scale character of the rock formation, which is constituted by porous domains dissected by the network of fractures. Taking into account the well-documented fact that porous rocks can be considered as a fractal medium and assuming that sizes of pores vary significantly (i.e. have different characteristic scales), the fractional-order differential equations that model the anomalous diffusive mass transport in such type of domains are derived and justified analytically. Analytical solutions of some particular problems of anomalous diffusion in the fractal media of various geometries are obtained. Extending this approach to more complex situation when diffusion is accompanied by advection, solute transport in a fractured porous medium is modeled by the advection–dispersion equation with fractional time derivative. In the case of confined fractured porous aquifer, accounting for anomalous non-Fickian diffusion in the surrounding rock mass, the adopted approach leads to introduction of an additional fractional time derivative in the equation for solute transport. The closed-form solutions for concentrations in the aquifer and surrounding rocks are obtained for the arbitrary time-dependent source of contamination located in the inlet of the aquifer. Based on these solutions, different regimes of contamination of the aquifers with different physical properties can be readily modeled and analyzed.

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### 1. Introduction

The common feature of all porous media is presence of two phases: solid phase (solid matrix) that occupies a part of domain and fluid phase, which occupies a void space in the rest of the domain. For example, the pore space of terrigenous rocks is a complex irregular system of communicating (but sometimes isolated) intergranular voids (pores) with sizes in the range from micrometers to tens of micrometers. Besides the granular porous medium which contains fluid in the intergranular pores space, in fractured rocks fluid flow takes place in a network of interconnecting fractures. A fractured porous medium is made up of blocks of an ordinary porous medium, possessing the nonzero porosity of blocks and high hydraulic conductivity of the network of fissures. Water flow and solute transport by the seeping groundwater are relatively slow and it is not possible to make experiments over the thousands of years and hundreds of meters of interest. Instead one has to rely on models that describe the processes and mechanisms that will be dominant over long times.

In order to deal with the flow and transport problems in fractured porous systems a number of modeling approaches were developed. These modeling approaches are traditionally divided into two major classes: discrete fracture models and continuum models. The first class is based on depicting the rock as a network of discrete fractures and another as a non-uniform, single, dual or multiple continuum. A third way of modeling is to combine these into a hybrid model of a non-uniform continuum containing a relatively small number of discrete dominant fractures. Extensive reviews of various approaches for modeling transport in fractured porous media (discrete models, continuum models and hybrid models) can be found in [43,44,8]. Although the discrete fracture network models are considered to be conceptually appealing approach (see, for example, [12,18,41,46,48,51,56]), in many situations a continuum concept is sufficient. For example, if fracture density is sufficiently high so that statistical methods can be employed, the system is likely to behave like a continuum. Furthermore, even for rather sparse fracture distribution, the continuum model with proper calibration can provide rather accurate tool for modeling the aquifer as it is shown in [19]. It is worth noting that continuum models are more convenient for practical applications, since these models demand less field data for calibration than discrete fracture network models [8,19].

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